**Monthly Payment and Loan Balance**

Many readers, for reasons of their own, want to know how to calculate the monthly payment and loan balance on amortized mortgages. Here are the formulas:  
  
The following formula is used to calculate the fixed Monthly Payment (M) required to fully amortize a Loan Amount (P) dollars over a term of n months at a monthly interest rate of r. [If the quoted APR is 6%, for example, then r is .06/12 or .005].

M = P [r (1 + r) n] / [(1 + r) n - 1]

M = Monthly Payment

P = Principal Loan Amount

r = interest rate per month

n = number of payments (months)

Javascript function:

M = P \* r / (1 - (Math.pow (1 / (1 + r), n)))

M \* (1 - (Math.pow (1 / (1 + r), n))) = P \* r

P = M \* (1 - (Math.pow (1 / (1 + r), n))) / r

P = M \* (1 - (Math.pow (1 / (1 + r), n))) / r

The next formula is used to calculate the remaining loan balance (B) of a fixed payment loan after p months.

B = P [(1 + r) n - (1 + r) p] / [(1 + r) n - 1]

Where:

* M = Monthly Payment
* P = Loan Principal
* r = Monthly Interest Rate (6% APR is .06/12 or .005)
* n = Loan Term in number of months
* p = Number of months paid
* Bn = Balance in month n

**Annual Percentage Rate (APR)**

Other readers ask about the formula used to calculate the APR. The APR is what economists call an "internal rate of return" (IRR), or the discount rate that equates a future stream of dollars with the present value of that stream. In the case of a home mortgage, the formula is

L - F = P1/(1 + i) + P2/(1 + i)2 +… (Pn + Bn)/(1 + i)n

Where:

* c = Interest rate per payment period (6% APR is .06/12 or .005)
* L = Loan amount
* P = Monthly payment
* n = Month when the balance is paid in full
* Bn= Balance in month n
* i = IRR
* F = Points and all other lender fees

This equation can be solved for i only through a series of successive approximations, which must be done by computer. Many calculators will also do it provided that all the values of P are the same.  
  
The APR is a special case of the IRR, because it assumes that the loan runs to term. In the equation, this means that n is equal to the term, and Bn is zero.   
  
If there is a monthly mortgage insurance premium, that premium must be included in P for as long as the balance exceeds 78% of the original property value. If there is an upfront premium, it is included in F. If the upfront premium is financed, P should be calculated based on the larger loan amount, but L should not include the premium.  
  
Note that on ARMs, the payments used to calculate the APR are those that would occur under the assumption that the index rate does not change over the life of the loan.  
  
On a cash-out refinance, the APR ignores the existing mortgage that is paid off, which makes it a poor guide to the decision (see [The APR on a Cash-Out Refinance](https://www.mtgprofessor.com/A%20-%20Mandatory%20Disclosure/apr_on_cashout_refi.htm)). The better guide is a "net-cash APR", in which the balance of the existing loan (including interest accrued to the day of payoff) is subtracted from the left side of the equation, and the "Ps" represent the difference in payment between the old and new mortgage.

**Future Values**

Many of my calculators measure financial results in terms of "future values" -- the borrower's net wealth at the end of a specified period.  
  
The future value of a single sum today is:

FVn = S(1+c)n

Where:

* FVn is the value of the single sum after n periods  
  S is the amount of the single sum now  
  c is the applicable interest rate  
  n is the length of the period

The future value of a series of payments of equal size, beginning after one period, is:

FVn = P[(1+c)n - 1]/c

Where P is the periodic payment, and the other terms are as defined above.

Calculate the amount of time required to pay off the loan using the following formula:

L = -log (1 – ((r \* P) / m)) / log (1 + r)

* L is the number of monthly payments required to pay off the loan. This is the number you are trying to calculate.
* r is the monthly interest rate.
* P is the amount of the loan principal balance.
* m is the monthly payment amount (this would include extra payment amount)

Log is the mathematical logarithm that requires a scientific calculator or Excel spreadsheet to calculate.

Example:

You have a $80,000 mortgage balance remaining with a 6 percent APR and plan to pay $1,600 per month until it is paid off.

The monthly interest is 6 / 100 / 12 = 0.005.

L = -log (1 - 0.005 \* $80,000 / $1,600) / log (1 + 0.005)

L = -log (1 - 0.25) / log (1.005)

L = -log (0.75) / log (1.005)

Any log base will give the same answer. This calculation uses base 10. N = 0.12494 / 0.00216 N = 57.84 This means it would take 57 months plus a smaller payment to pay off this mortgage. Divide by 12 to convert this to years. 57.84 / 12 = 4.82 years

JavaScript log()

Math.log(2); // Return the natural logarithm of the number “2”

L = - (Math.log(1 – ((r \* P) / (M))) / (Math.log(1 + r)))